

Academic Council

Item No: \_\_\_\_\_

Devrukh ShikshanPrasarakMandal's

**Nya.TATYASAHEB ATHALYE ARTS, Ved. S.R. SAPRE  
COMMERCE & Vid. DADASAHEB PITRE SCIENCE  
COLLEGE, DEVRUKH [AUTONOMOUS]**



**Syllabus for F.Y. B.Sc.**

**Program: B.Sc.**

**Course: Mathematics**

**Credit Based Semester and Grading System with the  
Effect from  
Academic Year 2019-20**

**B. Sc. General (Semester Pattern) B. Sc. First Year**

**MATHEMATICS – CURRICULUM**

Semester	Paper Code	Paper	Lectures /Practicals	Marks			Credits
				External	Internal	Total	
Semester I	ASPUSMT101	Theory Paper I - Calculus I	45	70	30	100	02
	ASPUSMT102	Theory Paper II - Algebra I	45	70	30	100	02
	ASPUSMTP01	Practical Paper I	09	35	15	50	01
Semester II	ASPUSMT201	Theory Paper I - Calculus II	45	70	30	100	02
	ASPUSMT202	Theory Paper II - Algebra II	45	70	30	100	02
	ASPUSMTP02	Practical Paper II	09	35	15	50	01

### Semester I Theory Paper I

**Learning Objectives:**

The students will be able to understand-

- Outline the concepts of limits and continuity.
- Analyze the properties of continuous functions.
- Identify differentiable functions.
- Analyze properties of differentiable functions.

Course Code ASPUSMT 101	Title	Lectures	Credits
<b>Unit</b>	<b>Calculus I</b>	<b>45</b>	<b>02</b>
<b>Unit I</b> Limit and continuity of functions of one variable	<p>1. Definition of Limit <math>\lim_{x \rightarrow a} f(x)</math> of a function <math>f(x)</math> evaluation of limit of simple functions using the <math>\epsilon - \delta</math> definition, uniqueness of limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand-limit <math>\lim_{x \rightarrow a^-} f(x)</math> right hand limit <math>\lim_{x \rightarrow a^+} f(x)</math> non existence of limits , <math>\lim_{x \rightarrow -\infty} f(x)</math>, <math>\lim_{x \rightarrow \infty} f(x)</math> and <math>\lim_{x \rightarrow a} f(x) = \infty</math>.</p> <p>2. Continuous functions: Continuity of real valued function on a set in terms of limits, examples, continuity of a real valued function at end points of domain, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity. Intermediate value theorem and it's applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.</p>	15	
<b>Unit II</b> Differentiability of functions of one variable	<p>1. Differentiation of real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable, first principal of derivative, algebra of derivative.</p> <p>2. Differentiable functions are continuous but not conversely, chain rule ,Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples)</p>	15	
<b>Unit III</b> Applications of differentiation	<p>1. Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave , convex , concave functions, points of inflection.</p> <p>2. Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing</p>	15	

	and decreasing function, examples,		
	3. L-Hospital rule without proof, examples of intermediate forms, Taylor's theorem with Lagrange's form of remainder with proof. Taylor's polynomial and applications		

### Semester I Theory Paper II

#### Learning Objectives:

The students will be able to understand-

- Learner will be able to experiment with divisibility of integers.
- Learner will be able to explain concept of functions and equivalence relations.
- Learner will be able to explain the properties of polynomials over  $\mathbb{R}$  and  $\mathbb{C}$ .

Course Code ASPUSMT 102	Title	Lectures	Credits
<b>Unit</b>	<b>Algebra I</b>	<b>45</b>	<b>02</b>
<b>Unit I</b> Integers & Divisibility	<ol style="list-style-type: none"> <li>1. Statements of Well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property. Binomial theorem for non-negative exponents, Pascal Triangle.</li> <li>2. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. Such as existence and uniqueness of g.c.d. of integers <math>a</math> &amp; <math>b</math> and that the g.c.d. can be expressed as <math>ma + nb</math> for some <math>m, n \in \mathbb{Z}</math>, Euclidean algorithm, Primes, Euclid's lemma. Fundamental Theorem of arithmetic, The set of primes is infinite</li> <li>3. Congruence, definition and elementary properties, Euler's <math>\phi</math> function, statements of Euler's theorem, Fermat's theorem and Wilson theorem, Applications.</li> </ol>	15	
<b>Unit II</b> Functions and Equivalence relations	<ol style="list-style-type: none"> <li>1. Definition of function, domain, co-domain and range of a function, composite functions, examples, direct image <math>f(A)</math> and inverse image <math>f^{-1}(B)</math> for a function <math>f</math>, injective, surjective, bijective functions. Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely examples of functions including constant, identity, projection, inclusion. Binary operation as a function, properties, examples.</li> <li>2. Equivalence relation, Equivalence classes, properties such as two equivalence classes are either identical or disjoint, definition of partition, every partition gives an equivalence</li> </ol>	15	

	relation and vice versa.		
	3. Congruence is an equivalence relation on $\mathbb{Z}$ , Addition modulo $n$ , Multiplication modulo $n$ , examples.		
<b>Unit III</b> Polynomials	<p>1. Definition of a polynomial, polynomials over the field where <math>\mathbb{F} = \mathbb{Q}, \mathbb{R}</math> or <math>\mathbb{C}</math>, Algebra of polynomials, degree of polynomial, basic properties.</p> <p>2. Division algorithm in <math>[ ]</math> (without proof), and g.c.d of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.</p> <p>3. A polynomial of degree <math>n</math> over <math>\mathbb{F}</math> has at most <math>n</math> roots, Complex roots of a polynomial in <math>\mathbb{R}[x]</math> occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree <math>n</math> in <math>\mathbb{C}[x]</math> has exactly <math>n</math> complex roots counted with multiplicity, A non constant polynomial in <math>\mathbb{R}[x]</math> can be expressed as a product of linear and quadratic factors in <math>\mathbb{R}[x]</math>, necessary condition for a rational number to be a root of a polynomial with integer coefficients, simple consequences such as <math>\sqrt{2}</math> is an irrational number where <math>p</math> is a prime number, roots of unity, sum of all the roots of unity.</p>	15	

Course Code ASPUSMTP01 Semester I Practical Paper I – Calculus I			
Sr.No.	Practicals	L	Cr
		30	01
1.	Properties of continuous and differentiable functions. Limits of functions using- " $\epsilon$ - $\delta$ " definition, (Simple functions recommended in the syllabus).		
2.	Algebra of continuous function.		
3.	Higher order derivatives, Leibnitz theorem, Mean value theorems and its applications.		
4.	Extreme values, increasing and decreasing functions. Applications of Taylor's theorem and Taylor's polynomials.		
5.	Miscellaneous Theoretical Questions based on full paper		

Course Code ASPUSMTP01 Semester I Practical Paper II – Algebra I			
Sr.No.	Practicals	L	Cr
		30	01
1.	Mathematical induction Division Algorithm and Euclidean algorithm in $\mathbb{Z}$ primes and the Fundamental theorem of Arithmetic. Convergence and Eulers function, Fermat's little theorem, Euler's theorem and Wilson's theorem,		
2.	Functions ( direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions. Equivalence relation.		
3.	Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.		
4.	Miscellaneous Theoretical Questions based on full paper.		

### Semester II Theory Paper I

#### Learning Objectives:

The students will be able to understand-

- Learner will be able to explain the properties of real numbers.
- Learner will be able to explain the notions of convergent sequences
- Learner will be able to test the convergence of series

Course Code ASPUSMT 201	Title	Lectures	Credits
<b>Unit</b>	<b>Calculus II</b>	<b>45</b>	<b>02</b>
<b>Unit I</b> Real Number System	1. Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$ , absolute value $  $ and its properties. 2. AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property. 3. Bounded sets, statements of I.u.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals.	15	
<b>Unit II</b> Sequences	1. Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences. 2. Convergence of standard sequences like $\left(\frac{1}{1+na}\right)$ $\forall a > 0$ , $(b)^n \forall 0 < b < 1$ , $\left(c^{\frac{1}{n}}\right) \forall c > 0$ and $\left(n^{\frac{1}{n}}\right)$ ,	15	

	<p>3. Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of <math>(1 + \frac{1}{n})^n</math></p> <p>4. Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse.</p>		
Unit III Series	<p>1. Series <math>\sum_{n=1}^{\infty} a_n</math> of real numbers, simple examples of series, Sequence of partial sums of a series, convergent series, divergent series. Necessary condition : <math>\sum_{n=1}^{\infty} a_n</math> converges <math>\Rightarrow a_n \rightarrow 0</math> , but converse is not true, algebra of convergent series,</p> <p>2. Cauchy criterion, divergence of harmonic series, convergence of <math>\sum_{n=1}^{\infty} \frac{1}{n^p}</math> (<math>P &gt; 1</math>), comparison test, limit comparison test, alternating series, Leibnitz's theorem (alternating series test) and convergence of <math>\sum_{n=1}^{\infty} \frac{(-1)^n}{n}</math>, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), root test (without proof) and examples.</p>	15	

### Semester II Theory Paper II

#### Learning Objectives:

The students will be able to understand-

- Learner will be able to experiment with the system of linear equations and matrices.
- Learner will be able to identify vector spaces.
- Learner will be able to explain properties of vector spaces and subspaces.
- Learner will be able to construct basis for a given vector space.
- Learner will be able to explain properties of linear transformation.

Course Code ASPUSMT 202	Title	Lectures	Credits
Unit	Algebra II	45	02
Unit I System of Equations and Matrices	1. Parametric Equation of Lines and Planes , System of homogeneous and non homogeneous linear Equations, The solution of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for ( m, n) = (1,2), (1,3), (2,2), (2,2), (3,3); Definition of n-tuple of real numbers, sum of n-tuples and scalar multiple of n-tuple.	15	

	<p>Deduce that the system of <math>m</math> homogeneous linear equations has a non trivial solution if <math>m &lt; n</math>.</p> <p>2. Matrices with real entries; addition, scalar multiplication of matrices and multiplication of matrices, transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrices, symmetric matrix, skew symmetric matrix, invertible matrix; Identities such as <math>(AB)^t = B^t A^t</math>, <math>(AB)^{-1} = B^{-1} A^{-1}</math></p> <p>3. System of linear equations in matrix form, Elementary row operations, row echelon matrix, Gaussian elimination method.</p>		
<p><b>Unit II</b> Vector Spaces</p>	<p>1. Definition of real vector space, Examples such as <math>\mathbb{R}^n</math>, <math>\mathbb{R}[x]</math>, <math>M_{m \times n}(\mathbb{R})</math> space of real valued functions on a non empty set.</p> <p>2. Subspace: definition, examples: lines, planes passing through origin as subspaces of <math>\mathbb{R}^3</math> respectively; upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrix as subspaces of <math>M_n(\mathbb{R})</math> (<math>n = 2, 3</math>); <math>p_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n</math>, <math>a_i \in \mathbb{R}</math>, <math>\forall 1 \leq i \leq n</math> as subspace of <math>\mathbb{R}[x]</math>, the space of all solutions of the system of <math>m</math> homogeneous linear equations in <math>n</math> unknowns as a subspace of <math>\mathbb{R}^n</math>.</p> <p>3. Properties of a subspace such as necessary and sufficient conditions for a non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is the subset of other.</p> <p>4. Finite linear combination of vectors in a vector space; linear span <math>L(S)</math> of a non empty subset <math>S</math> of a vector space, <math>S</math> is generating set for <math>L(S)</math>, <math>L(S)</math> is a vector subspace of <math>V</math>; Linearly independent/ Linearly Dependent subsets of a vector space, a subset <math>\{v_1, v_2, \dots, v_k\}</math> is linearly dependent if and only <math>\exists \epsilon \in \{1, 2, \dots, k\}</math> such that <math>v_\epsilon</math> is a linear combination of other vectors <math>v_j</math>'s.</p>	<p>15</p>	
<p><b>Unit III</b> Basis of a Vector Space and Linear Transformation</p>	<p>1. Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, any two basis of a vector space have same number of elements, any set of <math>n</math> linearly independent vectors in an <math>n</math> dimensional vector space is a basis, any collection of <math>n+1</math> vectors in an <math>n</math>-dimensional vector space is linearly dependent.</p>	<p>15</p>	



	<p>2. Extending any basis of a subspace <math>W</math> of a vector space <math>V</math> to a basis of the vector space <math>V</math>.          If <math>W_1</math> and <math>W_2</math> are two subspaces of a vector space <math>V</math> then <math>W_1 + W_2</math> is a subspace.  <math>\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)</math>.</p> <p>3. Linear Transformations: Kernel, Image of a Linear Transformation <math>T</math>, Rank <math>T</math>, Nullity <math>T</math>, properties such as: kernel <math>T</math> is a subspace of domain space of <math>T</math> and <math>\text{Img } T</math> is a subspace of codomain subspace of <math>T</math>. If <math>V = \{v_1, v_2, \dots, v_n\}</math> is a basis of <math>V</math> and <math>W = \{w_1, w_2, \dots, w_n\}</math> any vectors in <math>W</math> then there exists a unique linear transformation <math>T: V \rightarrow W</math> such that <math>T(v_j) = w_j = \forall 1 \leq j \leq n</math>, Rank nullity theorem (statement only) and examples.</p>		
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Course Code ASPUSMTP02 Semester II Practical Paper I – Calculus II			
Sr.No.	Practicals	L	Cr
		30	01
1.	Application based examples of Archimedean property, intervals, neighbourhood. Consequences of l.u.b axiom, infimum and supremum of sets.		
2.	Calculating limits of sequences, Cauchy sequences, monotone sequences.		
3.	Calculating limit of series, Convergence tests.		
4.	Miscellaneous Theoretical Questions based on full paper.		

Course Code ASPUSMTP02 Semester II Practical Paper II – Algebra II			
Sr.No.	Practicals	L	Cr
		30	01
1.	Solving homogeneous system of $m$ equations in $n$ unknowns by elimination for $(m,n)=(1,2),(1,3),(2,2),2,3,(3,3)$ , row echelon form. Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.		
2.	Examples of Vector spaces, Subspaces		
3.	Linear span of a non-empty subset of a vector space, Basis and Dimension of Vector Space		
4.	Examples of Linear Transformation, Computing Kernel, Image of a linear map, Verifying Rank Nullity Theorem		
5.	Miscellaneous Theoretical Questions based on full paper		

## Reference Books

1. R.R.Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.
3. T.M.Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.
4. Richard Courant- Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
5. Ajit Kumar- S.Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
6. Ghorpade, Sudhir R, -Limaye, Balmohan V, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
7. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
8. G.B.Thomas, Calculus, 12 th Edition 2009
9. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
10. Norman L.
11. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
12. Serge Lang, Introduction to Linear Algebra, Second edition Springer.
13. S. Kumaresan , Linear Algebra , Prentice Hall of India Pvt limited .
14. K.Hoffmann and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi, 1971
15. Gilbert Strang , Linear Algebra and it's Applications, International Student Edition.
16. L. Smith , Linear Algebra, Springer Verlang
17. A. RamchandranRao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.

## Evaluation Pattern

External evaluation: Internal evaluation (70:30)

Theory:-External evaluation (70 Marks) Question Paper Pattern

Time: 2.5 hours

No.	Question Pattern	Marks
Q.1	Fill in the blanks by choosing appropriate options (5 MCQs)	10
Q.2	a) Long Answer Questions (based on Unit I)	15
	b) Short Answer Questions (based on Unit I)	
Q.3	a) Long Answer Questions (based on Unit II)	15
	b) Short Answer Questions (based on Unit II)	
Q.4	a) Long Answer Questions (based on Unit III)	15
	b) Short Answer Questions (based on Unit III)	
Q.5	b) Long Answer Question (based on Unit I, II & III)	15
<b>Total</b>		<b>70</b>

Theory:-Internal evaluation (30 Marks)

Description	Marks
Test	10
Assignment	10
Overall Conductance	10
<b>Total</b>	<b>30</b>

Paper pattern for each course : ASPUSMT101, ASPUSMT102 and ASPUSMT201, ASPUSMT202

Practical:-Internal evaluation (40 Marks) Question Paper Pattern

No.	Question Pattern	Marks
Q.1	Fill in the blanks by choosing appropriate options (8 MCQs)	24
Q.2	Long Answer Questions (based on Unit I)	16
	Performance in Regular Practical's	05
	Viva	05
<b>Total</b>		<b>50</b>